SGI Varia; 102

SIMPLIFIED ANALYSIS OF SOIL-FOUNDATION-STRUCTURE INTERACTION

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December 1982

CONTENT

SGI nr 196 Klintland Grafiska, Linköping

SUMMARY

The issue of soil-foundation-structure interaction covers a broad and complex field in geotechnical engineering. Recently some international symposia on the topic have been held. The subject has been investigated by many researchers from many countries. The present paper deals only with the soil-foundation-structure interaction under static loading and includes investigation on long buildings of different kinds of structures. A simplified analysis of the problem is introduced and the variation of the bending moment of the foundation-structure system considered as a beam depending on the length of building, the kind of structures, the soil deformation properties and the non-homogenity of the soil are offered.

With the help of some practical examples, designers can choose a reasonable distance between deformation gaps and the area of steel for longitudinal strengthening bands to protect against damages of the building due to differential settlement.

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ACKNOWLEDGEMENTS

This report which was written at the Swedish Geotechnical Institute (SGI), Sweden in December 1982 is about some results from the studies on soil-structure interaction problems which have been carried out at the Institute for Building Science and Technology (IBST), Vietnam.

The writers would like to express their great thanks to Dr Jan Hartlén, Director of SGI and Dr Nguyen Manh Kiem, Director of IBST for their encouragement and interest in the project.

Special thanks to Prof. Nils-Erik Wiberg, CTH for his valuable recommendations and discussions.

Grateful thanks to Mr Per-Evert Bengtsson for his critical reading of the manuscript and his discussions.

Grateful thanks to Mr Pham Mao for his contribution during the study at IBST.

Gratitude is expressed to Mrs Eva Dyrenäs for her expert typing of the manuscript and to Mrs Rutgerd Åbrink for drawing the figures.

Finally the writers wish to express their sincere thanks to their colleagues at SGI and IBST for their help and encouragement with the project.

Linköping, December 1982

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List of symbols

b	=	width of the beam
$C(x)$	=	base stiffness
C	=	minimum value of the function $c(x)$ at two edges of the beam
C_{max}	=	maximum value of the function $c(x)$ in the middle of the beam
C_{av}	=	average value of the base stiffness
EI	=	equivalent flexural stiffness of the foundation-structure system considered as a beam
f_a	=	cross area of steel in all strengthening band at one floor level
h	=	height of a storey
$k(x)$	=	coefficient of subgrade reaction
K	=	number of storeys
l	=	half the length of the beam
l_M	=	half the length of the building for which the value of max M_{max} appeared
$M(\xi)$	=	bending moment of the beam
M_{max}	=	maximum value of the function $M(\xi)$
M_b	=	maximum moment that can be taken by the longitudinal strengthening bands
$\max M_{max}$	=	maximum value of function $M_{max}=f(l)$, see Fig 3
$p(x)$	=	soil reaction as load per length
q	=	uniform load on the beam
$Q(\xi)$	=	shear force in the beam
R_a	=	allowable tensile stress in steel
x	=	distance, see Fig 1
$y(x)$	=	vertical displacement of the beam
y_1	=	constant, see (11)
α	=	base stiffness gradient
δ	=	the ratio between C_{max} and C
ξ	=	x/l, non-dimensional distance

1. INTRODUCTION

A large number of buildings in Vietnam are founded on the alluvial ground of the Red River Delta and the Cuu-long River Delta. Here, soil conditions are very complicated, the soils are nonhomogeneous and highly deformable. This feature has a great influence upon the behaviour of structures founded in this area. There are two ways to limit the influence:

1. Improving the soil properties or using pile foundations which means that the subsoil has a small deformation under the load of the superstructure.
2. Designing the superstructure-foundation system so that it has a sufficient stiffness to prevent dangerous stresses which are caused by differential settlement.

In many cases the second way is more economic. For this purpose, the soil-foundation structure interaction problem has to be solved. Many scientists from various parts of the world have been interested in this problem: Meyerhof G. (1947), Chamecki S. (1956), Grasshoff H. (1957), Sommer H. (1965), Heil H. (1969), Larnach W.I. & Wood L.A. (1972), Lee I.K. (1979,1975), Beigler S.E. (1976)... Many effective numerical methods, especially the finite element method (FEM), have been developed rapidly. A large number of soil-structure interaction problems have been solved. Meanwhile, it is difficult for designers to use such results in design. A numerical solution is hardly used, for example, to design a building. Moreover, numerical solutions with a high accuracy are sometimes useless because of crude information about the soil conditions. In many cases, an analytical solution using some simplyfied assumptions will be useful and practical for designers. In this paper we introduce a simplified analysis of soil-foundation structure interaction problem.

2. ESSENTIAL ASSUMPTIONS AND GENERAL SOLUTION

It is very common that a building has a small width compared with its length. In this case the whole superstructure foundation system can be considered as a beam with an equivalent flexural stiffness EI. The value of the equivalent stiffness in a cross section of the building is theoretically equal to the bending moment on the cross section which causes a unit angular displacement $\theta = 1$. The way to determine the equivalent stiffness EI is shown in /11/. From the calculation results for four- to six-storey buildings of a large range of structure kinds (longitudinal frame structure or transversal frame with longitudinal strengthened band structure, with or without filled walls; brick masonry with longitudinal strengthened bands, prefabricated panel structure....) with a width of 8 to 10 m, the value of EI is about $1.0 \div 5.0 \times 10^7$ kNm^2.

For the soil reaction, the Winkler's approach is accepted:

$$p(x) = b \cdot k(x) \cdot y(x) \qquad (1)$$

where p(x) = load per length on soil under the foundation
y(x) = vertical displacement of the beam
b = width of the beam
k(x) = the coefficient of subgrade reaction

Later, we will call $c(x) = b \cdot k(x)$ - base stiffness. The value of c(x) varies along the structures, Fig 1a.

On a nonhomogeneous base the beam can be bent in some ways shown in Fig 2. The case in Fig 2a is the most common. But in this paper the calculation is made in two most dangerous cases shown in Fig 2b and c with a uniform load q.

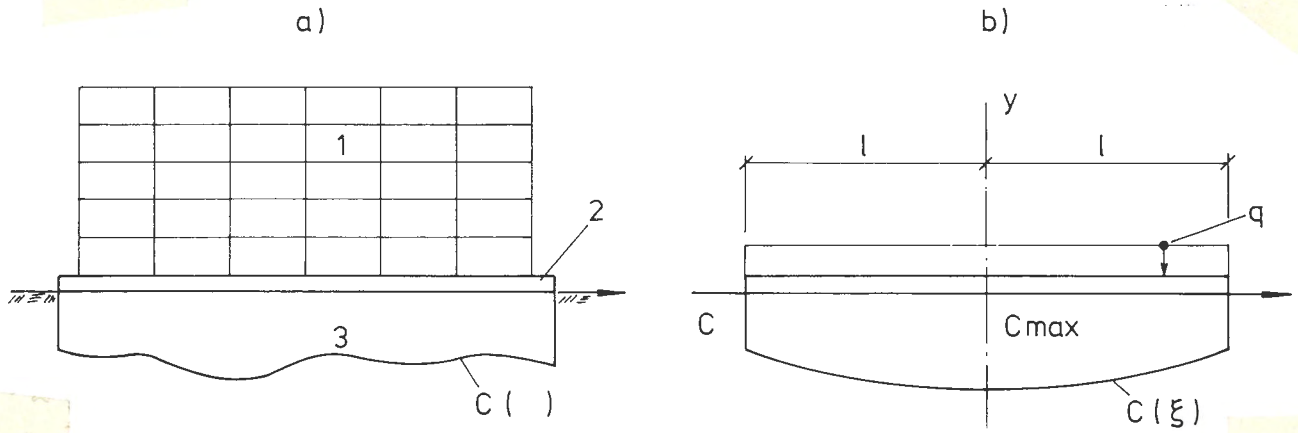

Fig 1. a) real system: 1. superstructure, 2. footing, 3. soil; b) equivalent beam on elastic foundation.

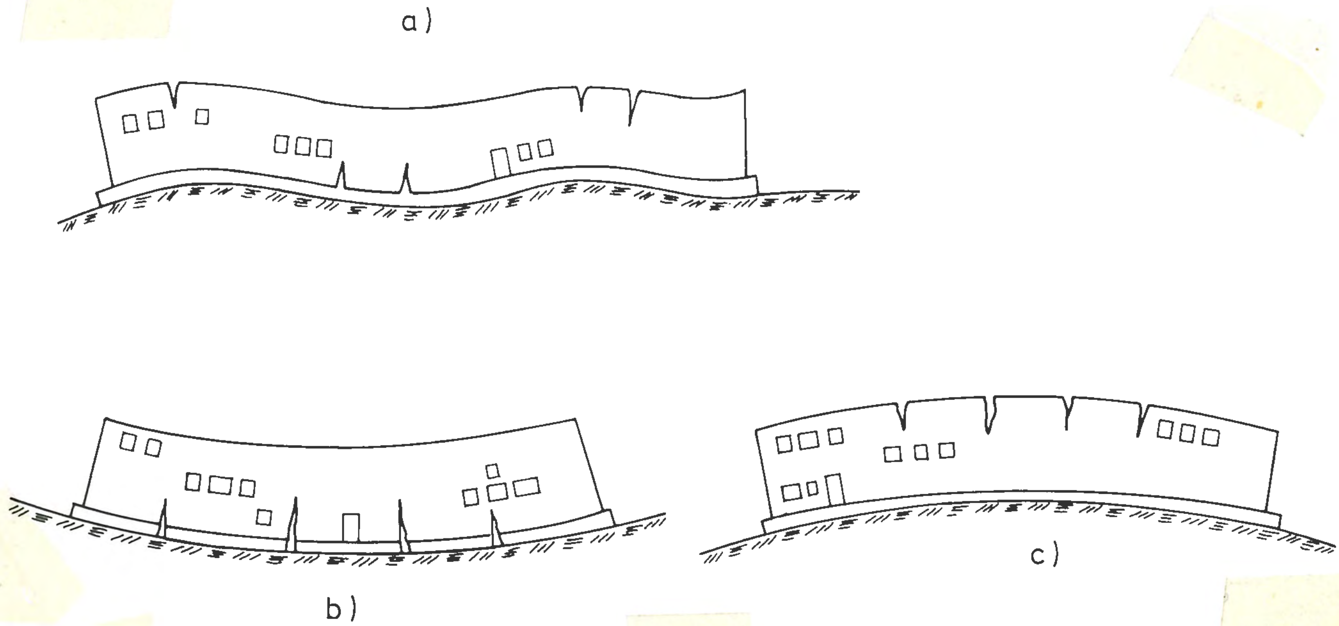

Fig 2. Differential settlement of building. a. general case, b. relative sag, c. relative hog.

And in the case shown in Fig 2b, for example, the variation of the C-value is:

$$C(\xi) = C\left[(1-\delta)\xi^2+\delta\right] \quad (2)$$

where $\xi = \frac{x}{l}$, nondimensional abscissa, Fig 1b

l = half of the length of the beam

δ = the ratio between the maximum value of the base stiffness C_{max} in the middle of the beam and the minimum value at the two edges of the beam.

We have also the following relation:

$$C = \frac{3\ C_{av}}{1+2\delta} \quad (3)$$

where C_{av} = the average value of the base stiffness. The non-homogeneous deformation properties of the soil is expressed by the base stiffness gradient:

$$\alpha = \frac{dC}{dl} \cong \frac{\Delta C}{\Delta l} \quad (4)$$

This means that the deformation properties of the soil is expressed by two values:

- the average value of the base stiffness C_{av}
- the base stiffness gradient α

With soil investigation in some points, the values of C_{av} and α can be determined.

Considering that the α value is constant and equal to the average value:

$$\alpha = \frac{C_{max} - C}{l} \quad (5)$$

we have

$$\delta = 1 + \frac{\alpha l}{C} \quad (6)$$

Referring to (4)

$$\delta = \frac{3C_{av} + \alpha l}{3C_{av} - 2\alpha l} \quad (7)$$

the expression (2) is rewritten against C_{av} and α :

$$C(\xi) = \frac{3C_{av}}{1+2\left(\frac{3C_{av}+\alpha l}{3C_{av}-2\alpha l}\right)} \left[\left(1-\frac{3C_{av}+\alpha l}{3C_{av}-2\alpha l}\right)\xi^2 + \frac{3C_{av}+\alpha l}{3C_{av}-2\alpha l}\right] \qquad (8)$$

The differential equation of a beam on Winkler's elastic foundation (when the shear deformation of the beam is neglected) is:

$$\frac{EI}{l^3} \cdot \frac{d^4\, y(\xi)}{d\,\xi^4} + C(\xi)\cdot y(\xi) = q \qquad (9)$$

in which y = vertical displacement of the beam
l = half of the length of the beam
q = uniform load on the beam

This equation can be approximately solved with some of the variational methods. Using Sobolev's solution, e.g. (by Galerkine's variational method) /13/, finally we have the bending moment and the shear force:

$$M(\xi) = -\frac{30\;EI}{l^2}\, y_1(\xi^2-1)^2 \qquad (10a)$$

where

$$y_1 = \frac{\left(4.14 - \frac{151+110\delta}{21(1+2\delta)}\right) \cdot \frac{q}{C_{av}}}{\frac{1}{1+2\delta}\left[59.2+27.2\delta\right] + \frac{366\;EI}{C_{av}\;l^4} - \left[\frac{151+110\delta}{21(1+2\delta)}\right]^2} \qquad (11)$$

and

$$Q(\xi) = -\frac{120\;EI}{l^3}\, y_1\xi(\xi^2-1) \qquad (10b)$$

From (10a), (10b) referring to (7), the bending moment and the shear force can be calculated for given values of C_{av}, α, EI and q.

3. EXAMPLES

Example 3.1

An example is given for a common case in Vietnam. The load of the building is 600 kN/m (the building has 4 to 6 storeys). The average value of base stiffness C_{av} = 5000 kN/m^2. The calculation is carried out with different values of the equivalent stiffness EI = 1.0×10^7 kNm^2, EI = 3.0×10^7 kNm^2, EI = 5.0×10^7 kNm^2 and with different values of the base stiffness gradient α = 40, 60, 80, 100 and 120 kN/m^3. The calculation results are shown in Fig 3.

Example 3.2

Buildings of prefabricated frame stucture type with filled walls and diaphragms are investigated in some cases:

a) the stiffness of the foundation and the value of α are constant, the number of storeys of the building varies from 5 to 10, the length of the building varies from 19.2 m to 115.2 m (from 1 to 6 sections), Fig 4a.

b) the value of δ and the length of the building are constant, the number of storeys varies from 5 to 10 the stiffness of the foundation is in direct proportion to the number of storeys.

c) the stiffness of the foundation, the values of α and C_{av} are constant (α=150 kN/m^3, C_{av}=8000 kN/m^2), the length of the building varies from 19.2 m to 115.2 m and the number of storeys varies from 5 to 10, Fig 4b.

The equivalent flexural stiffness of the building is determined in the way shown in /11/. The results of the calculations are shown in Fig 4.

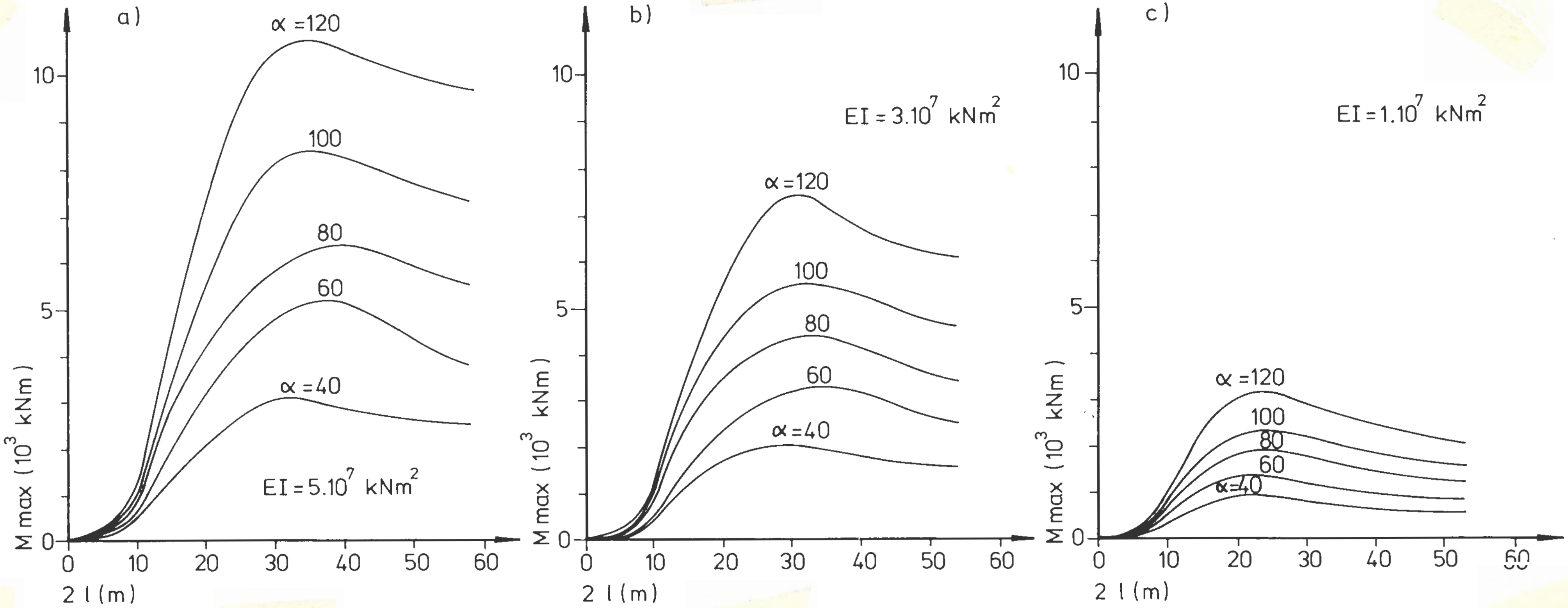

Fig 3. Relationship between the value of M_{max} and the length of the building with the value of α varying from 40 to 120 kN/m^3.
a) for EI = 5.10^7 kNm^2 b) EI = 3.10^7 kNm^2
c) EI = 1.10^7

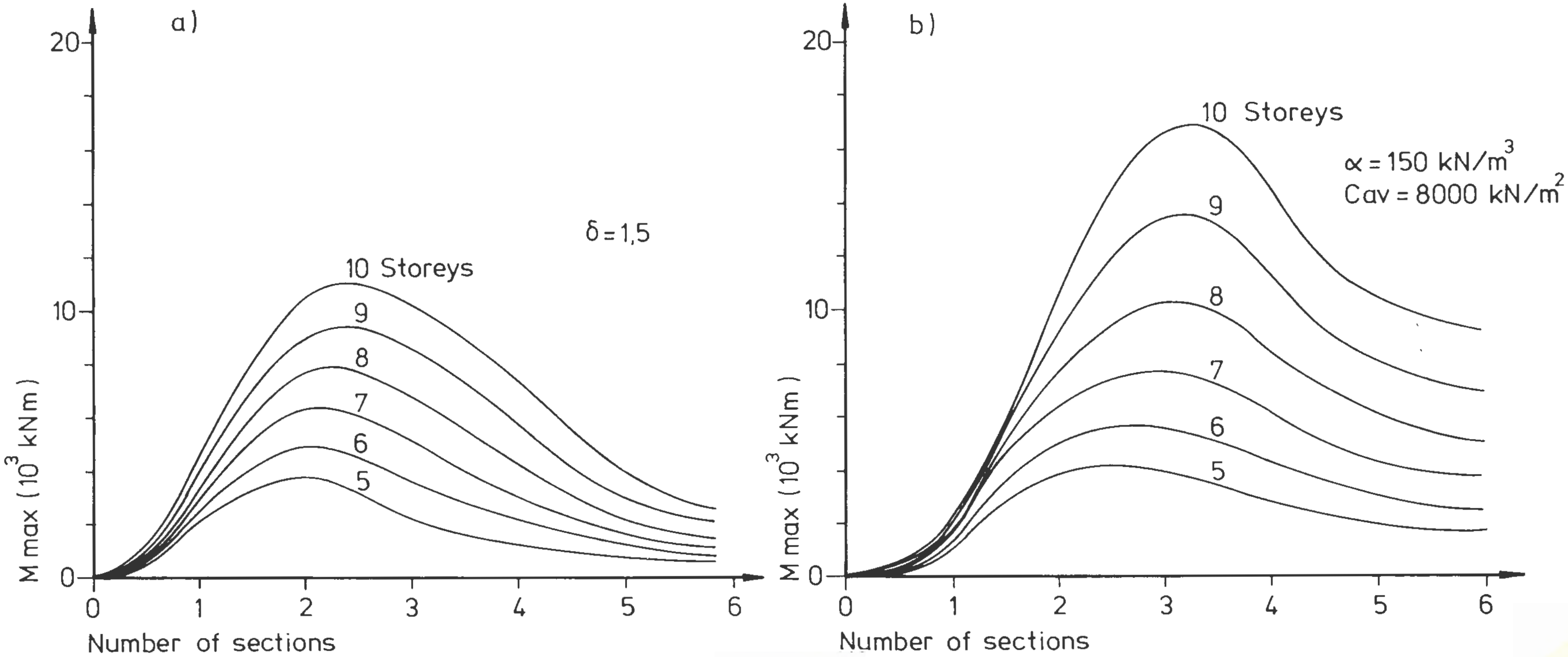

Fig 4. Relationship between the value of M_{max} and the length of building (here the length of one section L = 19.2 m) with the number of storeys varying from 5 to 10. a) for the case where δ = 1.5 = const, b) for the case where α = const, δ varying against the length of the building, C_{av} = 8000 kN/m^2 = const.

4. APPLICATIONS FOR DESIGN

4.1

For buildings not allowed to be cracked and founded on a complex soil, the best length of the building is 21 = 20 m. With increased length, the building should be divided by dilatation gaps. From the calculation results, though the values of α and EI covered a large range ($EI=1.10^7-5.10^7$ kNm², α = 40-120 kN/m²) the maximum bending moment, M_{max}, is rather small with a length of the building of about 20 m.

4.2

Accepting some simplified assumptions: a) the neutral axis is at the top level of the foundation, b) is a cross-section, tensile stress due to bending moment is taken by steel in longitudinal bands and strip foundation and is distributed in linear way, see Fig 5. The maximum bending moment which can be taken by the strengthening bands is called M_b and can be predicted by:

$$M_b = f_a R_a h \frac{1}{K} \sum_{i=0}^{K-1} (K-i) \qquad (12)$$

in which

f_a = cross area of steel in all strengthening bands at a floor level

R_a = allowable tensile stress in the steel

h = the height of a storey (in this case the height of every storey is the same)

K = the number of storeys

To prevent the building from damages due to differential settlements, the value of M_b has not to be less than the maximum bending moment M_{max} which is determined by the above-mentioned method when considering the building foundation system as a beam on elastic base.

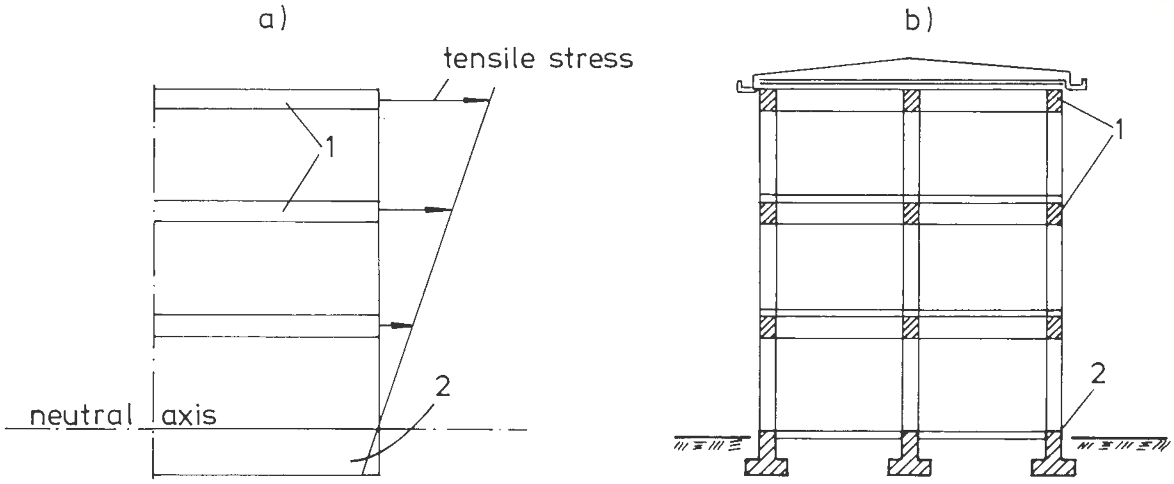

Fig 5. a) simplified assumptions; b) cross section of the building; 1) longitudinal bands, 2) strip footing.

There can be two designing problems:

- With a given value of M_b, chosing the suitable length of the building by drawing the diagram $M_{max} = f(l)$ and pointing out where $M_{max} \leq M_b$.
- With a determined length of building, chosing the way to distribute steel in strengthening bands so that M_b which is determined by expression (12) is not less than M_{max}.

Example. A five-storey building of prefabricated panel structure has a story height, which is the same for every storey, of 2.7 m. On each floor-level, there are longitudinal bands with a total cross-area of steel of 12 cm^2 and R_a = 10 kN/cm^2.

According to (12) M_b = 3560 kNm.

The equivalent stiffness of the building, which is determined according to /11/ is about $5.0x10^7$ kNm^2. From the diagram in Fig 3a, the length of the building can be chosen for various values of α.

Table 1

α, in kN/m^3	60	80	100	120
2l, in m	44	36	32	29

With the same longitudinal bands, but for a frame structure, the building has the calculated equivalent stiffness of about $3.0x10^7$ kNm^2. From the diagram in Fig 3b, the length of the building can be chosen for various values of α.

Table 2

α, in kN/m^3	80	100	120
2l, in m	42	34	32

4.3

With a constant equivalent stiffness of the building and a constant value of α, when the length of the building is increased, the value of M_{max} is limited by a certain value called max M_{max}. In the examples, the length of the building where the value of max M_{max} appeared is about 50 to 70 m and is called $2\ l_M$. If the superstructure is strengthened so that $M_b \geq$ max M_{max}, the building does not need to be divided by dilatation gaps and the length of the building can be greater than the value of $2\ l_M$ without any danger. For some types of buildings the amount of steel needed for each longitudinal band so that $M_b \geq$ max M_{max} is shown in Table 3 for different values of α.

Table 3

Type of building	Predicted value of EI	Steel area necessary for each longitudinal band, in cm^2 for some values α, in kN/m^3			
	kNm^2	60	80	100	120
Prefabricated large panel structure of 5- to 6-storeys Frame with continued longitudinal diaphragm on the whole length of the building	$5x10^7$	17	21	28	36
Prefabricated large panel structure of 3- to 4-storeys Frame structure of 5- to 6-storeys with interrupted longitudinal diaphragm Brick masonry of 4- to 5-storeys	$3x10^7$	16	21	28	36
Brick masonry of 3- to 4-storeys Frame structure of 4- to 5-storeys without longitudinal diaphragm	$1x10^7$	10	15	18	25

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5. CONCLUSIONS

This is a simple solution to the soil-foundation structure interaction problem. It helps designers to consider the rigidity of the superstructure in the calculation of its foundation settlement. It makes clear the influence of differential settlement on the behaviour of the structure. From this solutions, designers can choose a reasonable distance between deformation gaps, the area of steel for longitudinal strengthening bands to protect against damages due to differential settlement...

- If the rigidity of the structure is great enough, according to the results of the calculations, deformation gaps cannot be required.

- In the contrary case shown in Fig 2c the solution is similar to the case shown in Fig 2b and the strip foundations of the building is considered as longitudinal bands.

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