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Area Correction in Triaxial Testing

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March 1993

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Preface

In 1992, a triaxial testing of stabilized soil was performed At the Swedish Geotechnical Institute (SGI) in the scope of a large project related to the soft-soil improvement using lime, cement and lime-cement columns. As a guest researcher at SGI, the author was engaged to set up the testing apparatus, to define the testing procedure and to test the improved-soil specimens. This paper was prepared as a part of the work on that project and on an another project related to triaxial testing of soft clay at elevated temperatures.

Mensur Mulabdic′
Linköping, march 1993

Contents

Summary

The area correction in triaxial testing is one among several corrections that are normally applied in deviator stress calculations. It is determined by the form of the specimen after failure and by the changes in vertical and volumetric strain.

When testing hard soils (highly overconsolidated) and when using frictional ends, a single-plane failure surface or zone failure develops so that special area corrections have to be made.

The type of area correction recently proposed by La Rochelle at al. (1988) is analyzed and a new method, based on experience gained while testing improved soils, is proposed.

The proposed method is based on the principle similar to the one used by La Rochelle et al. (1988), but it combines pure sliding in direct form with the change in specimen diameter.

It was determined through separate analyses that the method proposed by La Rochelle et al. should be modified by correcting the diameter correction expression.Both methods, the one developed by La Rochelle et al. with suggested correction and the proposed one, provide similar results. The proposed model was applied in the analysis of two real test results. The results obtained show that the residual strength is greater and its associated vertical strain is smaller then the corresponding values obtained using simple area correction where only vertical and volumetric strains are considered.

The proposed correction can easily be applied to any type of failure characterized by the developed failure zone or rupture surface, provided that the inclination of the failure surface to the horizontal and the specimen diameter are measured at the end of testing.

1. Introduction

Several corrections of the computed shear stress are generally required when evaluating the triaxial test results: correction for filter papers, for friction on piston, for membrane influence as well as the correction for the cross-section area. This paper focuses on the area correction, while the special emphasis is placed on testing in cases when failure zone or single-plane (rupture) surface is formed. In fact, the paper concentrates on the post-failure state, when the so called residual strength develops on the plane of failure (rupture surface).

Triaxial testing is basically performed in two stages: consolidation and shearing. At both stages, specimen dimension changes are a consequence of changes in stress and/or general conditions (e.g. temperature) with respect to the in situ conditions. In order to correctly determine the deviator stress (i.e., the vertical stress) usually acting on the central plane of a specimen, it is necessary to establish the actual cross-section area. The deviator stress is caused by the piston-transferred load, which results in a vertical pressure greater than the cell pressure, after all corrections (e.g. friction) have been made. While it is relatively easy to define the load transferred to the specimen by the piston, it is much more difficult to determine the actual cross-section area of a specimen.

One way to determine it at any stage of testing is to constantly measure the change in specimen diameter (at the central part of specimen), which is not often the case in practice. The other more common method is to calculate the cross-section area from the measured volumetric and vertical deformation and initial area. In this "simple" approach, it is assumed that the change in the specimen cross-section area is uniform with height. That may be the case during consolidation and shearing when lubricated ends are used. However, this condition is almost never met when frictional ends are used. Depending on the specimen shape after shearing, it is possible to calculate the cross-section area under asumption that the diameter change is parabolic or that bulging occurred [3]. These methods provide reasonable results up to the failure point, or even further on, if there is no single-plane failure surface.

Nevertheless, the above methods are not suitable after the failure is attained, i.e. in cases where single-plane failure surface develops (usually when hard soils are sheared), or when a failure zone in the specimen has been formed. This happens when frictional ends are used, and especially when overconsolidated clays are submitted to slow testing. The failure surface appears after the failure and usually spreads from one edge cap to another, or at least touches one of them. In such cases it is necessary to apply other methods, taking into account the changes in the cross-section area around the failure zone and the change in size of the contact area between two specimens parts that slide one over another.

The area correction is normally made in two steps: the simple area correction (based on the change in vertical strain and volume) is used up to the peak strength, while a method similar to those considered in this text are used beyond that point.

Two of such methods are of interest: the method based on pure sliding of two specimen parts one over another and the observational method in which the shape of the cross section area at the end of testing and the sliding between two specimen parts are taken into account.

It is still a matter of discussion whether or not to proceed with the testing after having developed the rupture surface. Problems arise due to uncertainty in the stress-strain distribution around such zones. The residual strength developed on a single failure surface, that should in fact constitute the final goal of the analyses involving the cross-section area correction, is therefore questioned.

Despite such doubts regarding evaluation of the post-failure behavior around the rupture zone, this phenomenon was analyzed at SGI in the scope of the triaxial testing of clay stabilized with lime and/or cement.

The author of this text considers that the results of analyses presented in this paper may also be applied in cases of failure involving formation of failure zones (as recently proposed by other authors [5]), i.e. that the application should not be limited to the rupture zone only. This statement can be supported by the fact that such a failure zone could be imagined as a set of failure surfaces and that the load transfer over it has a pattern quite similar to that observed at the rupture surface. A subsequent triaxial testing of soft clay at elevated temperatures performed at SGI also resulted in failure zone and/ or failure surface formation. Tests were performed without lubricated ends, as a quick and slow undrained shearing, after anisotropic consolidation. The failure zone and the correction of its area should be considered since shearing is concentrated to a narrow zone so that the failure is not achieved simultaneously in all parts of the specimen. This paper was prepared as an attempt to help in solving this problem.

2. Description of the phenomenon

Specimens can assume different forms after triaxial testing, depending on the type and nature of soil, the type of testing and conditions related to the friction between the specimen and caps.

Some typical examples are shown in Fig. 1, together with the evaluated influences exerted on the deviator stress value.

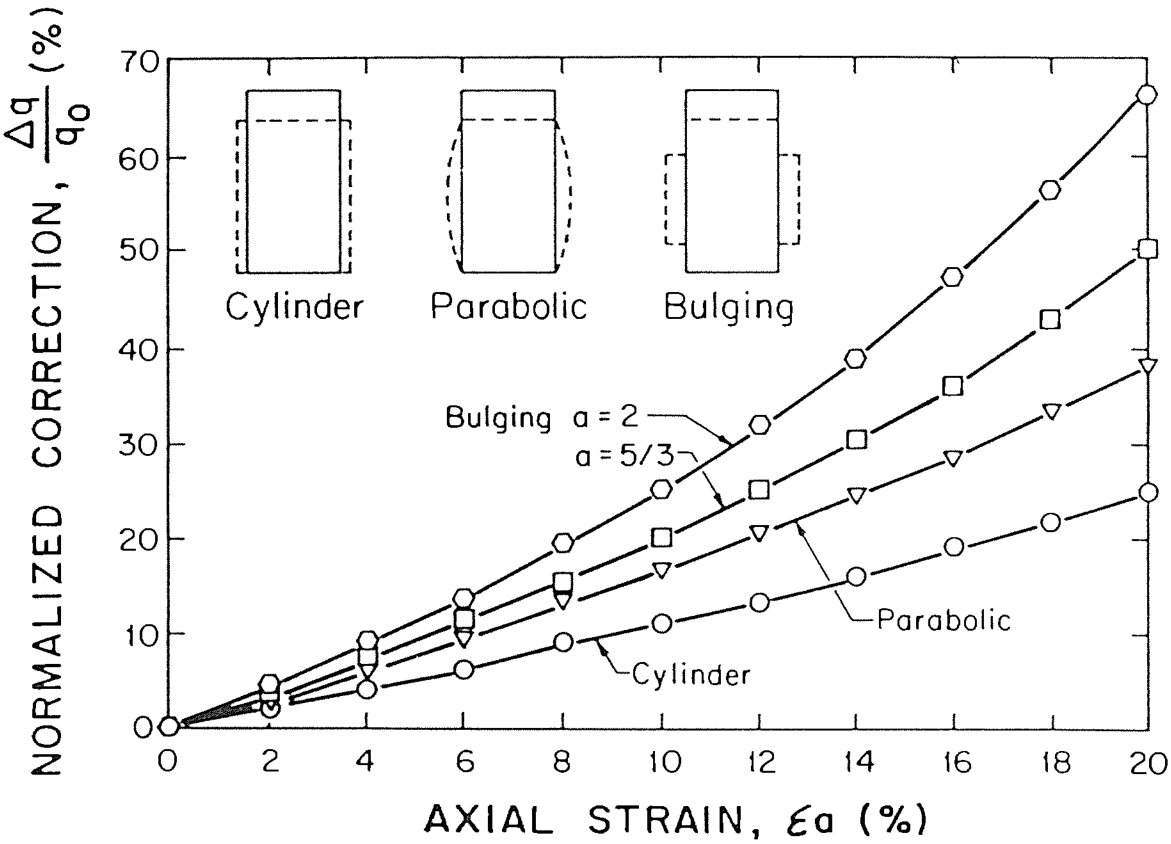

Fig. 1. Typical modes of area development for specimen submitted to triaxial testing, with relative influence on deviator stress at the constant volume compression (after [1]).

According to [1], the following expressions may be used for calculating area at different modes of deformation (compressive strains are positive):

cylindrical
$$A = A_0\left(\frac{1-\varepsilon_v}{1-\varepsilon_a}\right) \tag{1}$$

parabolic
$$A = A_0\left(-\frac{1}{4}+\frac{\sqrt{25-20\varepsilon_a-5\varepsilon_a^{\,2}}}{4(1-\varepsilon_a)}\right)^2 \tag{2}$$

bulging $$A = A_0\left(\frac{1-\varepsilon_v}{1-\alpha\varepsilon_a}\right) \qquad (3)$$

where compressive deformations are positive and where

A_0 = initial area at zero strain
a = experimental constant, normally between 1 and 2
ε_a = vertical strain
ε_v = volumetric strain

Expression (1) is a "simple" method, used when no better approximation is considered possible. It averages cross section area over the entire height and therefore overestimates vertical stresses in the central section where area is normally the greatest.

For the bulging mode, it is supposed that strains are concentrated in the central part of the specimen. For parabolic change in diameter, the area is calculated at the largest midplane section.

Different situation is when a failure zone develops together with the change in form, thus indicating concentration of shear strains and stresses at a certain area in the specimen. Such a case is presented in Figure 2. In this case, none of the previously defined expressions for area correction can be applied.

The situation is even more drastic when a single-plane failure surface develops, as shown on Fig.3. This is characteristic for highly overconsolidated soils, especially when tested with frictional ends. Pure sliding occurs after formation of the failure surface, when the contact surface decreases with the deformation. Some increase in diameter is associated with this mode, but normally not much.

Some other problems are encountered when the single-plane failure develops, like for instance membrane stretching and cavity formation. These phenomena are shown in Figure 4. According to [4], the analyses from published papers dealing with these aspects of the single-plane failure can be summarized as follows: membrane effects are mainly dependent on cell pressure, and it is not likely that the shear strength on the formed surface will be influenced by this effect. The influence is smaller then indicated in the idealized condition presented in Fig. 4. However, the decrease in volume will occur or the negative pore pressure will develop, and the pore pressure field around the surface will be influenced. If frictional ends are used, the cap will become inclined and the friction in the piston zone will be increased.

A series of triaxial tests was performed at the Swedish Geotechnical Institute using the laboratory-prepared specimens of soft soil improved by adding variable quantities of cement and/or lime. This testing was performed as a part of a greater project concentrating on soil improvement using lime, cement and lime-cement columns. Some specimens after failure are shown in Fig. 5.

The specimens made of silty clay improved by adding 10% of cement were tested 45 days after mixing. After anisotropic consolidation, they were sheared under drained conditions (specimen BCED160, effective cell pressure: 160 kPa, cf. Fig. 5.) as well as under undrained conditions (specimens BCEU160, BCEU020, effective cell pressure 160 kPa and 20 kPa, respectively) and the constant rate of deformation amounting to 0.0167 mm/min was applied.

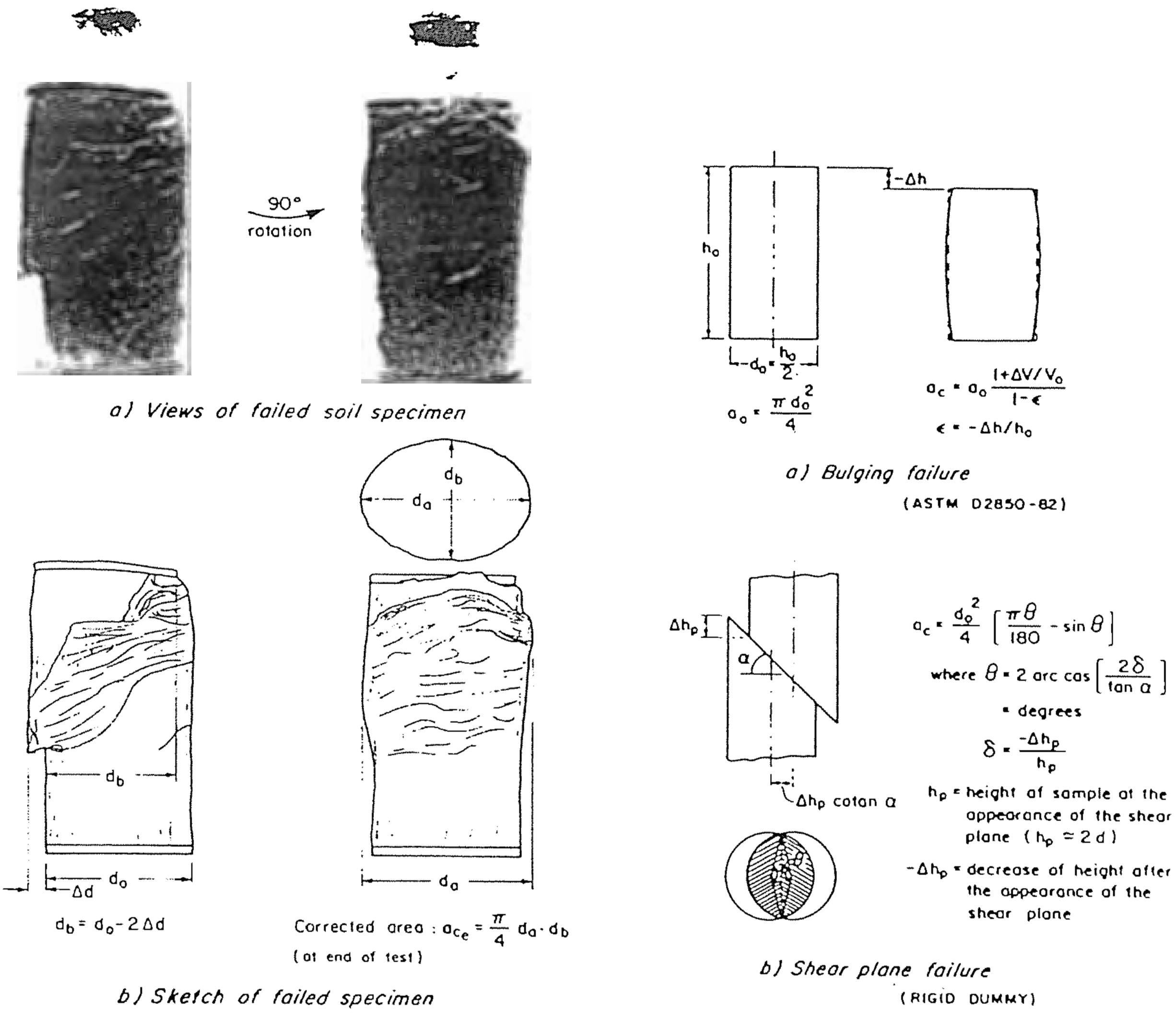

Fig. 2. Shear plane failure in a soil specimen (after [5]).

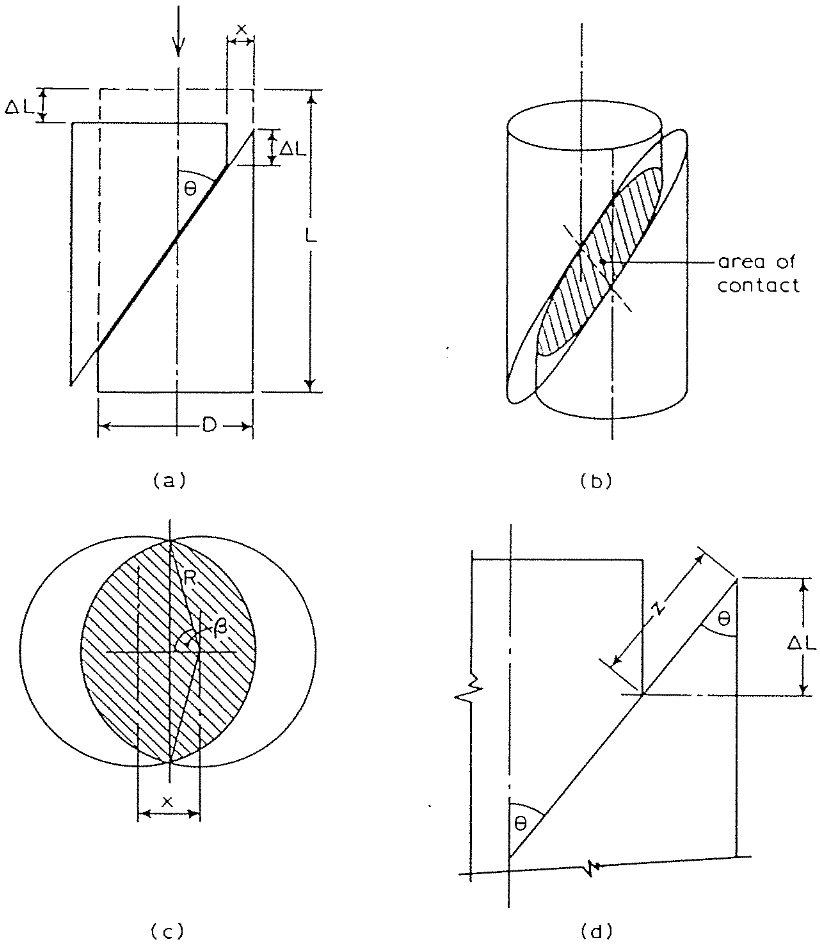

Fig. 3. Area correction due to single-plane slip:
(a) mechanism of slip,
(b) area of contact between two portions of the sample,
(c) projected area of contact, (d) displacement along slip surface as related to vertical deformation (after [4]).

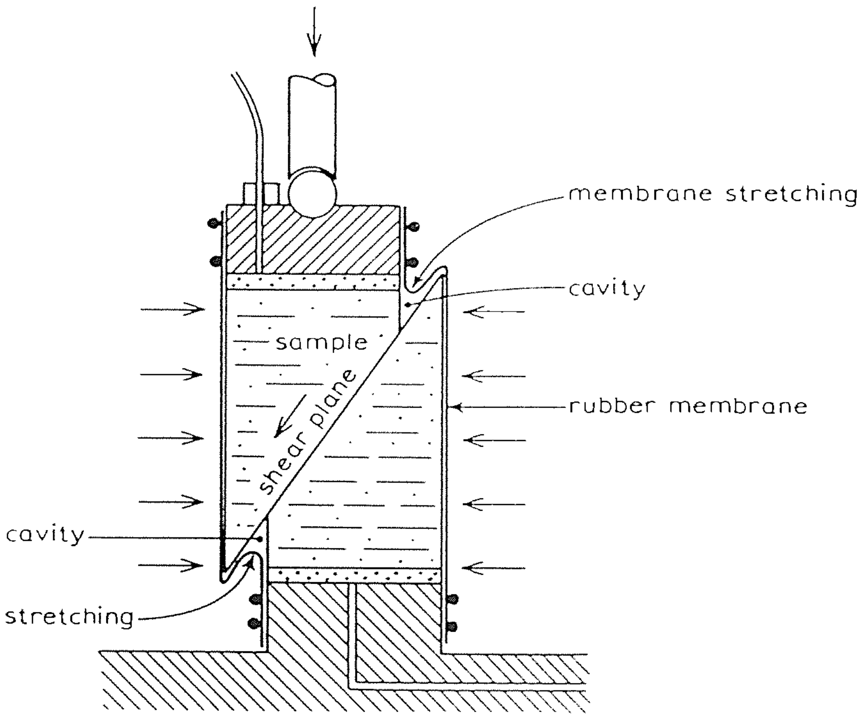

Fig. 4. Rubber-membrane stretching due to single plane surface (after [4]).

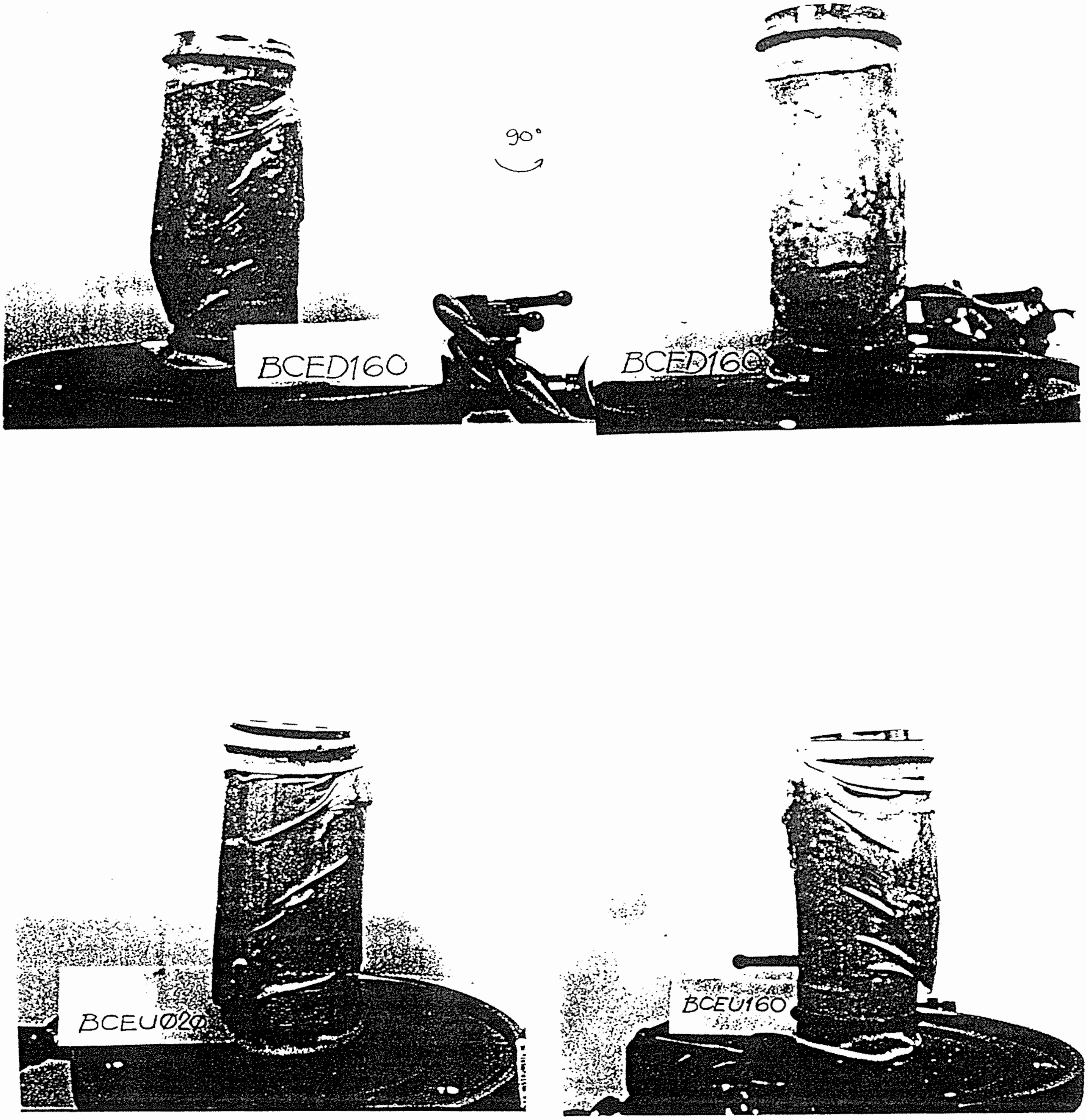

Fig. 5. Improved-soil specimens after failure (source: SGI's archives).

3. Area correction for failure zones

Two methods are widely used for area correction in case of single-plane failure/zone failure: the method based on the pure sliding of two parts of a specimen one over another and observational method proposed by La Rochelle at al. [5].

The latter method is based on the specimen form after failure and on the following assumptions:

- the cross section area after failure assumes an ellipsoidal form,
- a simple area correction is valid up to the failure point, while an elliptic area starts to develop after failure,
- the area after failure is proportional to the deformation developed after failure and to the change in area from the failure point to the end of testing,
- the sliding is taken into account by correcting greater diameter of ellipse for the specimen part that is out of contact.

Parameters used for calculating area according to [5] are presented in Fig. 2. The author considers that the expression for the d_b value shown in that figure (b) should be modified as follows:

$d_b = d_0 - \Delta d$ instead of $d_b = d_0 - 2\Delta d$

since for lateral displacement of Δd, the diameter of the contact area is smaller then the original one exactly for that amount, not for twice as much, as stated in [5], see Fig. 2. The following expression is used for calculating the actual area after failure (after [5])

$$a_c = a_f + (a_{ce} - a_f)\left(\frac{\varepsilon - \varepsilon_f}{\varepsilon_e - \varepsilon_f}\right) \qquad (4)$$

where

a_f = cross-section area at peak strength
a_{ce} = cross-section area at the end of testing (see Fig. 2)
ε_e = axial strain at the end of testing
ε_f = axial strain at peak strength
ε = axial strain after peak strength

According to the approach by [5], it is necessary to measure two diameters of the specimen after failure (d_a and d_b) as well as the size of the "missing contact" between two parts of the specimen (Δd). As will be seen in the following section, it is not always possible to measure it correctly due to the form of the failure zone (surface) and to bending at the corner of the upper part of the specimen that is not in contact with the lower part.

4. Proposed area correction

An attempt was made at SGI to clarify the area correction problem by analyzing a series of triaxial tests performed at SGI using different types of cohesive soil improved by adding lime, cement or lime and cement in various proportions. Specimens were compacted in laboratory and the characteristics obtained are summarized in table 1.

Table 1. Characteristics of the cement-stabilized soil.

soil type	water content %	density g/cm^3	unconfined strength kPa
clay 10%cement	66	1.57	225
silty clay 10%cement	25	1.92	1200
clay-gyttja 16%cement	120	1.32	440

Specimens 50 x 100 mm in size were prepared for the testing. They were first anisotropically consolidated and then sheared under undrained or drained conditions, with the constant vertical deformation rate of 0.0167 mm/min.

After shearing, the specimens were carefully examined and typical parameters were measured: d_a, d_b and Δd (as shown in Fig. 2).

It was established that:

- they fail in a single failure plane
- average inclination of the plane to the horizontal is approx. 61°
- the upper part of the lower portion, and the lower part of the upper portion of the specimen were disturbed; the former had vertical tension cracks while the latter was always bent downwards due to the membrane stretching and cell pressure that provoked some horizontal tension cracks.
- the diameter of the upper part was usually somewhat greater than that of the lower part.

It is possible to distinguish two effects that simultaneously occur during shearing: expansion of the specimen's diameter and sliding of its upper part over the lower part. These two effects were combined in the approach proposed by La Rochelle at al [5], thus letting the size of ellipse axes (d_a and d_b) and the size of "missing contact" (Δd) define the actual area of the specimen after failure point [Fig. 2.].

In the approach proposed by the author, it is assumed that the above two effects should be taken into account by modeling the following "sequence" of events : initially, the circular area having diameter equal to d_a develops and then pure sliding occurs between two parts of the specimen (having so defined diameter) as for the increment of vertical strain in excess of failure strain. This means that the diameter increases in increments from the one at failure defined through "simple" approach (as in [5]) to that at the end of testing, and the pure sliding is calculated for each increment value. This calculation should be performed for each vertical strain beyond the strain corresponding to the peak deviator stress value (failure point).

Consequently, the actual specimen diameter after the failure point should be calculated according to the following expression:

$$d_{(\varepsilon)} = d_f + (d_e - d_f)\frac{\varepsilon - \varepsilon_f}{\varepsilon_e - \varepsilon_f} \qquad (5)$$

where d and ε denote diameter and vertical deformation, respectively, the index "e" defines the end of testing, while "f" is the failure point.

The actual area after failure is then calculated as:

$$A = \frac{d^2_{(\varepsilon)}}{4}(\pi\frac{\theta}{180} - \sin\theta)$$

$$\theta = 2\arccos(\frac{2(\varepsilon - \varepsilon_f)}{\tan\alpha})$$

$$d_f = \sqrt{\frac{4}{\pi}(A_0\frac{1-\varepsilon_{vf}}{1-\varepsilon_f})} \qquad (6)$$

where :

- d_f = the diameter at failure of the lower part of specimen, opposite to the direction of shearing (type d_a)
- α = the inclination of the failure surface to the horizontal.
- A_0 = the area at zero strain
- ε_v = the volumetric strain
- ε = the vertical strain

5. Application of the suggested area correction

Several methods used for calculating cross section area have been compared and the results obtained are presented in Fig. 6. The comparison is based on values obtained through real CAD testing of silty clay blended with cement. The figure shows area development calculated according to the following methods: (1) "simple" calculation, (2) the method presented by La Rochelle at al. 1988. using the proposed 2Δd and Δd (the author considers that the use of Δd is more appropriate) as correction values for larger ellipse diameters, (3) the proposed method and (4) pure sliding. When the simple method is applied, the specimen cross section area constantly increases with the vertical strain, while an exaggerated contact area reduction is obtained using the pure sliding method.

The method presented by La Rochelle at al. [5] provides results that are relatively similar to those obtained using the proposed method, but only if Δd is used to correct greater ellipse diameters. If 2Δd is used for correcting greater ellipse diameters, as originally proposed by La Rochelle et al. [5], then the results would be similar to those obtained by pure sliding.

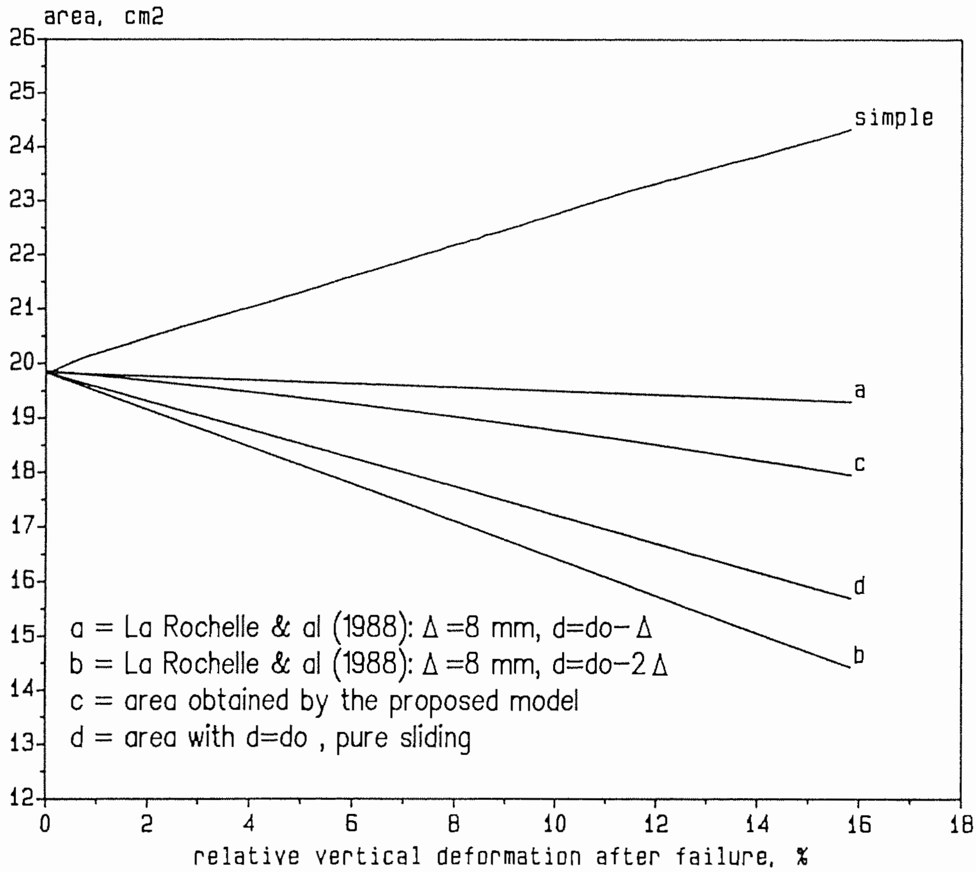

Fig. 6. Development of the vertical strain area calculated by different methods for testing silty clay soil containing 10 % cement (failure strain is 1.38 %).

If the value obtained by the simple calculation of area is normalized by all the values from other methods, then the obtained area ratios will correspond to those presented in Fig.7. The greatest difference was observed for La Rochelle at al. with 2Δd, while the proposed method and the method by La Rochelle at al. with the suggested modification differ significantly beyond the vertical strain which is by 6-8 % in excess to the failure deformation.

This difference is partly due to the inaccuracy in measuring the Δd value, and partly to the use of a different calculation principle.

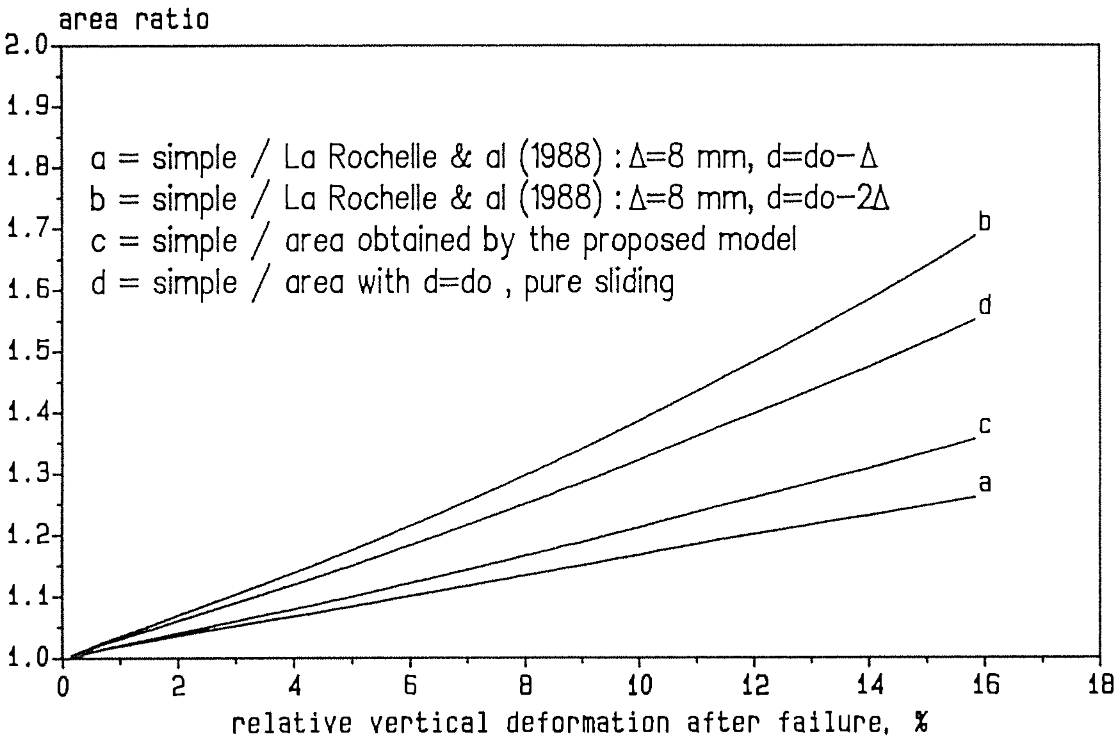

Fig. 7. Relationships between areas calculated by different methods.

The real result of the CAD triaxial testing performed on a silty clay specimen with 10 % cement is presented in Fig. 8. in terms of the deviator stress and vertical strain, calculated with area correction using the "simple" and the proposed approach. A significant reduction in strength with deformation (peak to residual strength) was observed. If the proposed approach is used, the residual value will be obtained earlier, i.e. at about 6-8 % of the vertical strain after failure, and will be by approx. 10 % greater than the corresponding value obtained by the simple area correction.

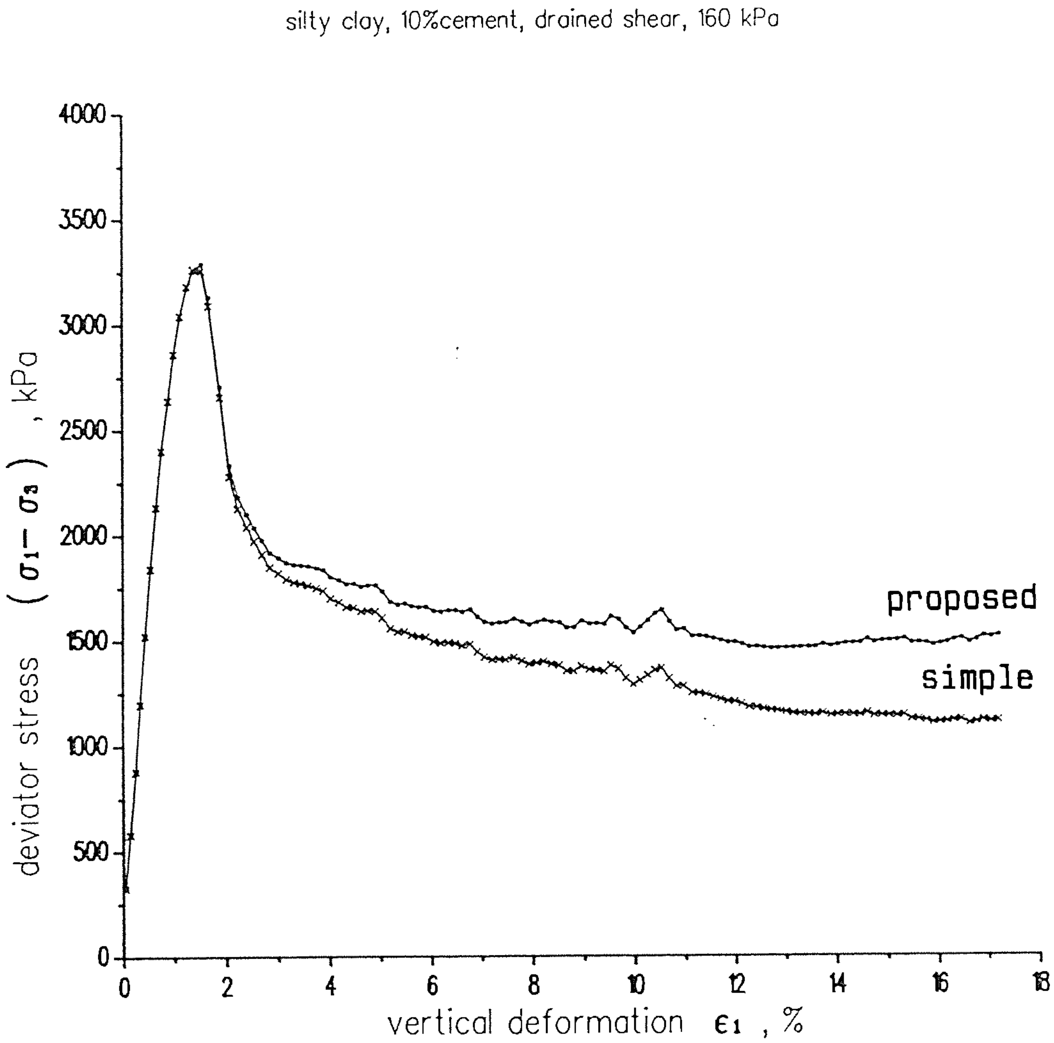

Fig. 8. Results of the CAD testing using silty clay with 10 % cement, interpreted by the standard and the suggested method.

A similar analysis, with similar conclusions, is presented in Fig. 9 for CAU testing performed on a silty clay specimen containing 10 % of the lime and cement. The increase in residual strength after vertical strain of 14 % is due to an additional resistance gained from O-ring on the top cap in contact with the lower portion of the specimen.

Photographs of sheared specimens tested in the scope of an another SGI project related to the soft-clay triaxial testing at elevated temperatures are presented in Fig. 10. Once again, we may observe development of the failure zone or rupture surface. The testing was performed using frictional ends.

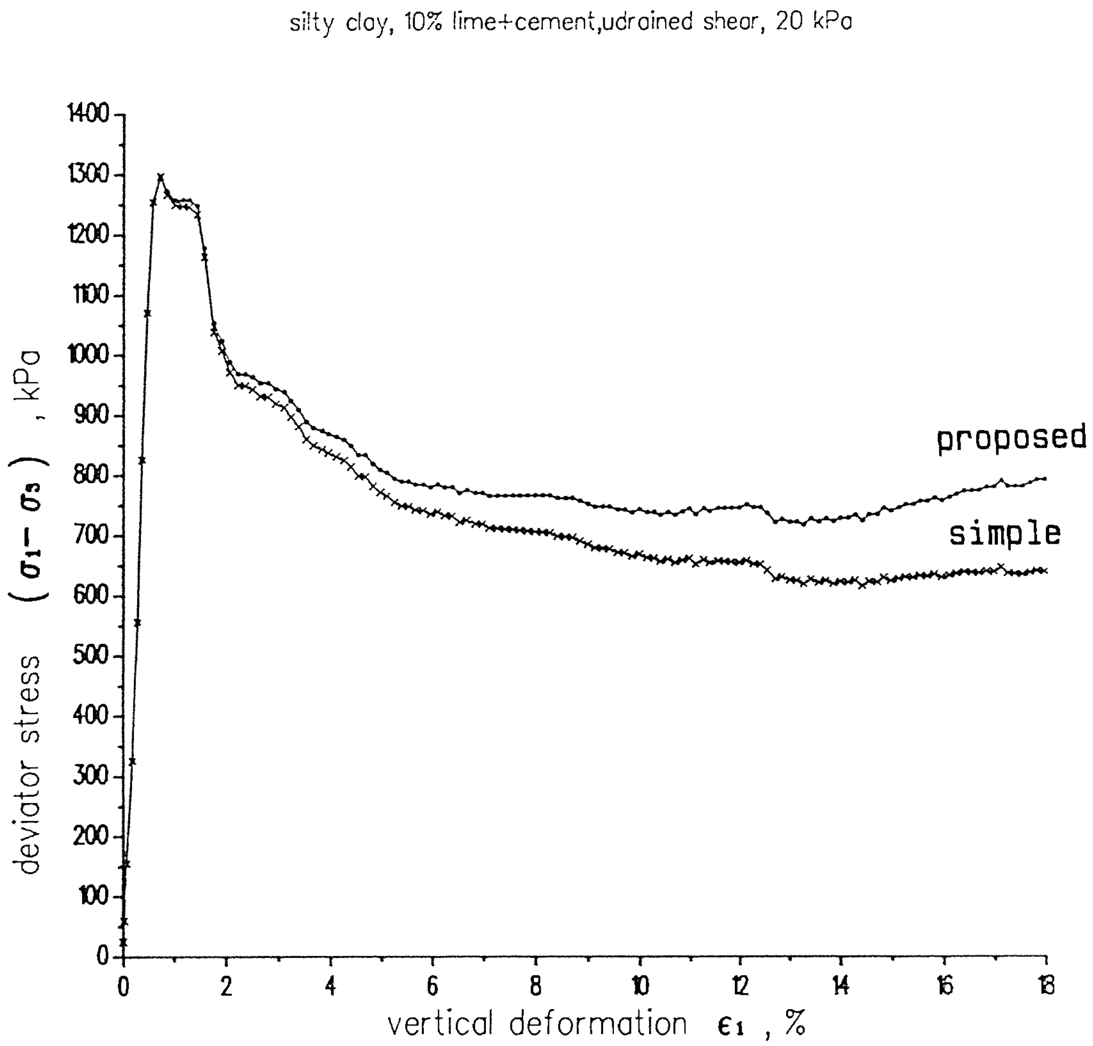

Fig. 9. **CAU test results using silty clay with 10 % lime + cement, interpreted by the "simple" and the suggested method.**

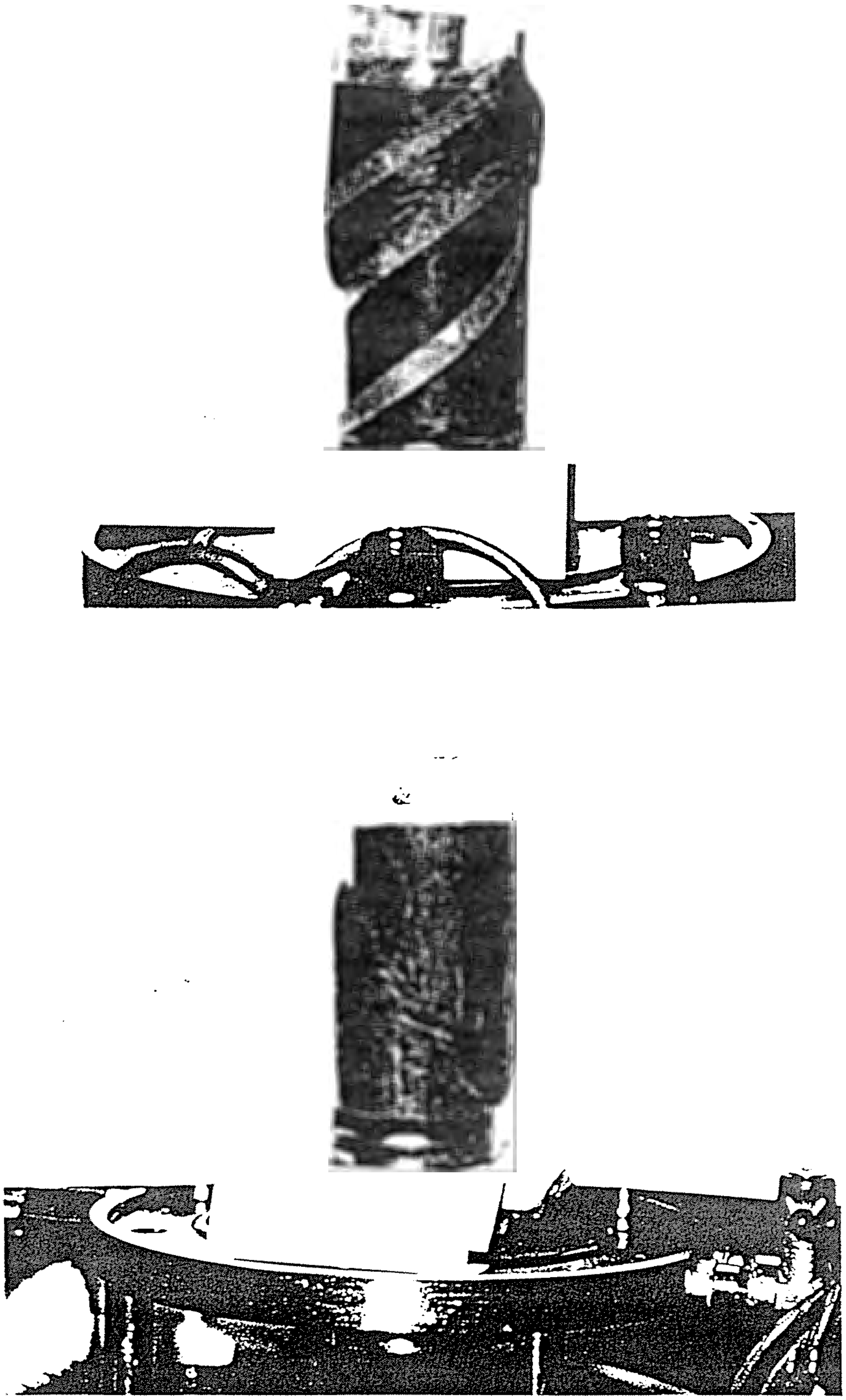

Fig. 10. Typical soft clay specimens sheared at elevated temperatures.

6. Conclusion

The cross section area correction during triaxial shearing of firm soils exhibiting single-plane failure or zone failure has been examined. Several existing methods of area correction are presented and the new one is proposed. The effects of different correction methods are illustrated by some examples, selected from a series of results obtained by testing soil improved by some additives (cement and/or lime). The analysis has led to the following conclusions:

- it is wise to correct cross section area according to the specimen form after testing (observational approach)

- the correction is to be linearly applied to post-failure conditions, depending on the change in area from the failure point to the end of testing

- a simple correction, based on the change in vertical and volumetric deformation, can be used up to the failure point

- the proposed correction and the one by La Rochelle at al. [5] but with suggested modification, provide similar results and may be recommended; a larger ellipse diameter for the latter should be calculated as d_b-Δd, i.e. not as d_b-$2\Delta d$ (Fig. 2)

- when recommended corrections are used, the residual strength is greater than the one associated with the simple correction; the difference increases with the vertical strain

- the residual strength of analyzed specimens was achieved at the vertical strain of 6-8%

- if recommended area corrections are used, it might be sufficient to run test up to the 10 % of vertical strain to obtain the residual value

- recommended corrections may be applied to failure types involving formation of a single-plane or failure zone

7. Literature

[1] Baldi,G.,Hight,D.W. and Thomas,G.E., "A reevaluation of Conventional Triaxial Test Methods", Advanced Triaxial Testing of Soil and Rock,ASTM STP 977, Robert T.Donaghe, Ronald C. Chaney and Marshall L.Silver, Eds., American Society for Testing and Materials, Philadelphia,1988, pp 219-263.

[2] Bishop,A.W. and Henkel,D.J., "The Measurement of Soil Properties in the Triaxial Test", Arnold, London,1962.

[3] Germaine,J.T. and Ladd,C.C.," Triaxial Testing of Saturated Cohesive Soils", Advanced Triaxial Testing of Soil and Rock, ASTM STP 977, Robert T.Donaghe, Ronald C.Chaney and Marshall L.Silver, Eds., American Society for Testing and Materials, Philadelphia, 1988,pp 421-459.

[4] Head,K.H.,"Manual of soil laboratory testing",Vol 3,Pentech Press,London,1986.

[5] La Rochelle,P.,Leroueil,S., Trak,B., Blais-Leroux,L. and Tavenas,F., "Observational Approach to Membrane and Area Corrections in Triaxial Tests", Advanced Triaxial Testing of Soil and Rock, ASTM STP 977, Robert T.Donaghe, Ronald C.Chaney and Marshall L.Silver,Eds., American Society for Testing and Materials, Philadelphia, 1988, pp 715-731.